Using Mathematics to make Cancer Treatments Safer

Source Optimisation for Brachytherapy Radiation Problems

Key Tool

PDE Constrained Optimisation

Methods

Forward Model

- Radiation Transfer Equation simplified to diffusion.

$$-\nabla \cdot (A \nabla u(x)) = f(x)$$

f : Source
u : Radiation Flux
A : Tissue densities

- Simple domain model: tumour(T) and non-tumour (Ω/T).
- Finite Element Method

Optimisation Process

- Cost Functional

$$J(u,f) = \frac{1}{2}\|u - d_T\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|f\|^2_{L^2(\Omega)}$$

- Dose constraint:

$$d_T = \begin{cases} 1, & \text{in } T \\ 0, & \text{in } \Omega/T \end{cases}$$

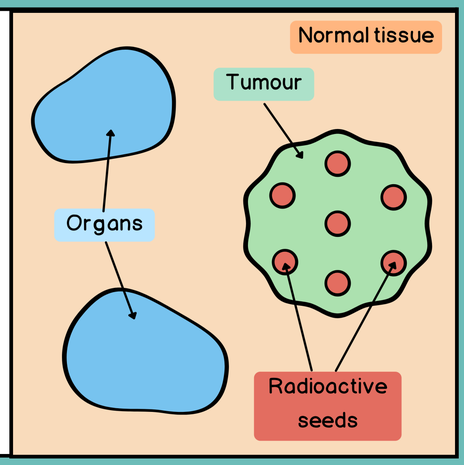
- Lagrangian method for optimality conditions.
- Gradient Descent.
- Output : 'ideal' source function.

The Problem

Brachytherapy: a type of cancer treatment where the radiation is implanted directly onto the tumour.

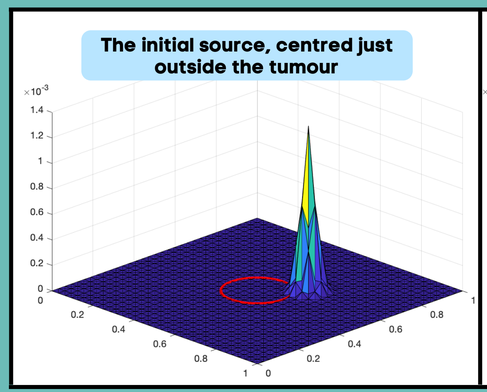
Issue: when the tumour is located close to critical organs, the radiation can damage them, causing further health complications.

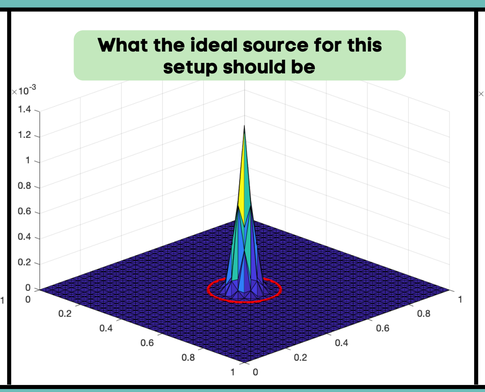
Where to place the radiation to minimise the damage to critical organs while still treating the tumour?

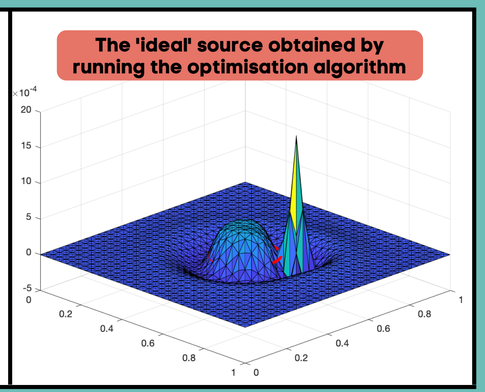


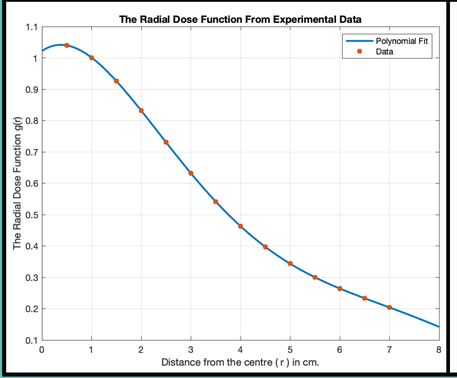
Results

Optimising the position of **one seed.** Unit square domain, with a circular tumour of radius 0.125 centred [0.5,0.5].









- The output f from the algorithm doesn't have the same structure as the initial f or the expected f for this problem, which is a Gaussian with narrow support.
- In reality, f is given by a **radial dose function** $g(r)$. Radiologists measured the radiation spread from the source and fit the data to a **quintic polynomial**

$$g(r) = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5$$

What's Next?

- The radiation source term has a certain shape that the current algorithm doesn't take into account.
- This shape would clearly give the position where the seed should go.
- The next step is to add shape constraints for f to this process.

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More Info!

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